Financial calculations

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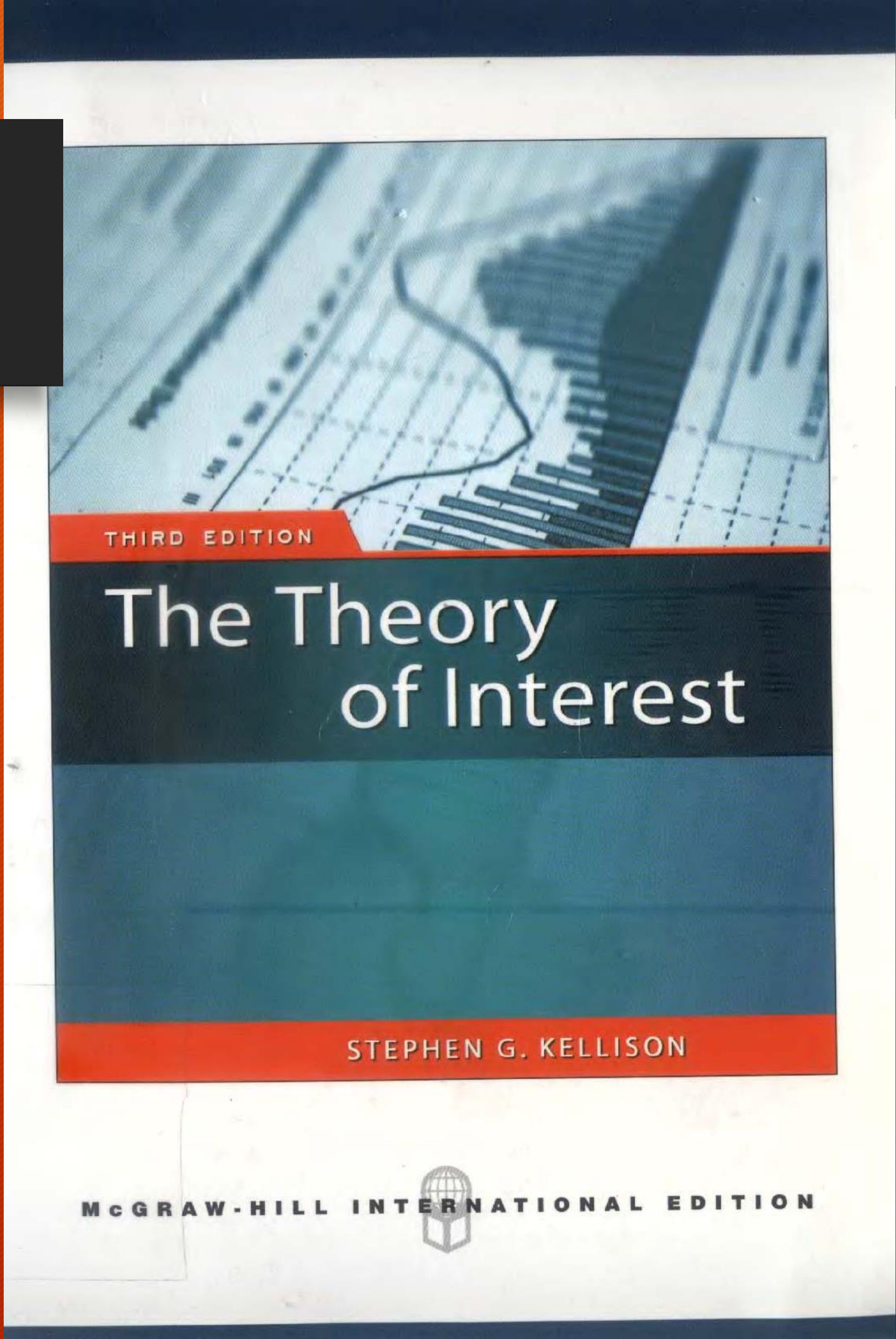
The theory of interest 1.

Which option would you choose from the following ones:

- *To get 1,000 USD now, or*
- *To get 1,000 USD in a year?*

Why?



The theory of interest 2.

- **Interest** is money paid by a bank or other financial institution to an investor or depositor in exchange for the use of the depositor’s money.
- Amount of interest is (usually) a fraction (**called the interest rate**) of the initial amount deposited called the **principal amount**.

FV calculation – simple interest

Notation:

- *FV* : amount due – Future Value
- *C* : principal amount – the money you invest
- *r, (i)* : interest rate (%)
- *t, (n)* : time (in years)

These quantities are related through the equation:

$$FV = C * (1 + r * t)$$

$$t = \frac{Duration\ of\ the\ investment\ (in\ days)}{Number\ of\ days\ in\ a\ year}$$

Determining (t)…

Method name	# of days / month	# of days / year
German	30	360
French	real	360
British	real	365
Real	real	365/366

Practicing (t) calculations:
01/01/2019-02/01/2019
01/01/2019-28/02/2019
05/02/2019-12/12/2019

FV calculation – compounded interest

Once credited to the investor, the interest may be kept by the investor, and may earn interest itself.

If the interest is credited once per year (*compounded*), then after (*t*) years the amount due is calculated by the following equation:

$$FV = C_0 * (1 + r)^t$$

$$r = \sqrt[t]{\frac{FV}{C_0}} - 1$$

$$t = \frac{ln\frac{FV}{C_0}}{\ln(1 + r)}$$

FV calculation – compounded interest 2.

If a portion of the interest is credited after a fraction of a year, then the interest is said to be compounded. If there are (f) compounding periods per year, then in (t) years the amount due is:

$$FV = C_0 * \left(1 + \frac{r}{f}\right)^{f*t}$$

The annual interest rate equivalent to a given compound interest rate is called the *effective interest rate (APR, Annualized Percentage Rate):*

$$r_e = \left(1 + \frac{r}{f}\right)^{f} - 1$$

FV calculation – continuously compounded interest

What happens as we increase the frequency of compounding?

The amount due for continuously compounded interest or effective interest rate is:

$$\boxed{FV = C_0 * e^{r*t}}$$

The *effective interest rate (APR) is*:

$$r_e = e^r - 1$$

PV calculation

Present value (PV), also called ‚discounted value' is the current worth of a future sum of money, or stream of cash flow given a specified rate of return. Future cash flows are discounted at the discount rate (r); the higher the discount rate, the lower the present value of the future cash flows.

Discrete compounding:

$$PV = \sum_{t=1}^{n} \frac{C_t}{\left(1 + \frac{r}{f}\right)^{t*f}}$$

$$DF_{r,n} = \frac{1}{(1+r)^n}$$

$$PV = C_t * \left(\frac{1}{1 + t * r}\right)$$

Continuous compunding:

$$PV = \frac{C}{e^{r*t}}$$

Required Rate of Return

Pequired Rate of Return (r) is depend on:

- Risk-free rate of interest
- Liquidity
- Riskiness of the investment

NPV calculation

Net Present Value (NPV) is the difference between the present value of cash inflows and the present value of cash outflows over a period of time. NPV is used in capital budgeting and investment planning to analyze the profitability of a projected investment or project.

$$NPV = -C_0 + \sum_{t=1}^{n} \frac{C_t}{(1+r)^t}$$

We can accept an investment if it's NPV >= 0

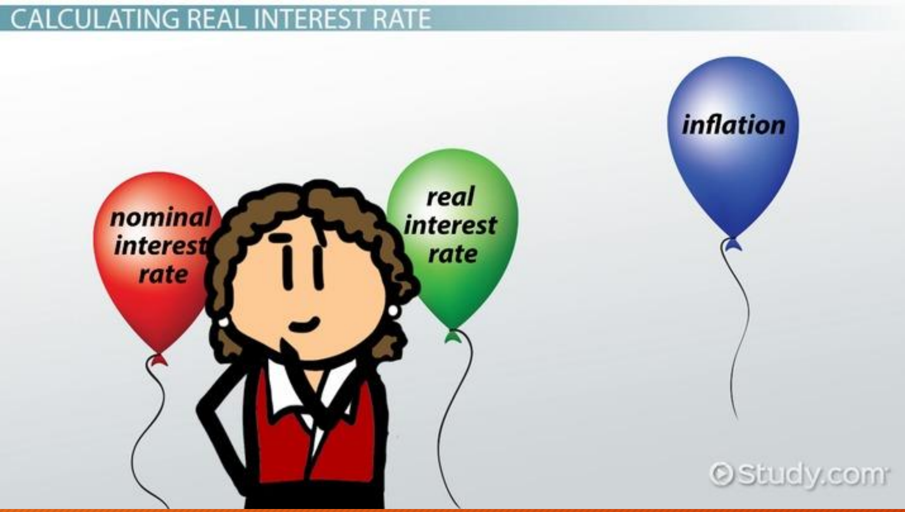
IRR calculation

Internal Rate of Return (IRR) on an investment or project is the ‚annualized effective compounded return rate' or rate of return that sets the net present value of all cash flows (both positive and negative) from the investment equal to zero.

(IRR formulas are shown at the examples)

We can accept an investment if it's IRR ≥ r

Real interest rate calculation



Real Interest Rate (r_r) A real interest rate is an interest rate that has been adjusted to remove the effects of inflation to reflect the real cost of funds to the borrower and the real yield to the lender or to an investor.

$$r_r = \left(\frac{1 + r_n}{1 + i}\right) - 1$$

Simplyfied calculation:
Real interest rate = Nominal interest rate – Inflation rate

Real interest rate calculation - Example

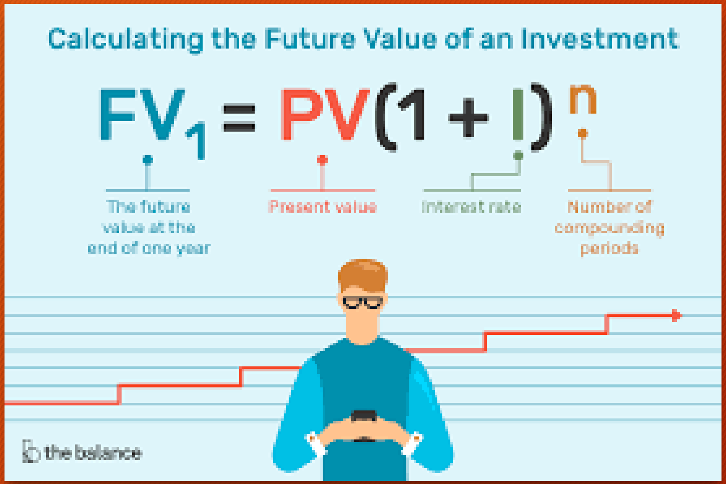
Nominal rate of interest is 5%. The inflation is 3%. What is the annual real rate of interest? What is the simplified real rate of interest?

$$r_r = \left(\frac{1 + r_n}{1 + i}\right) - 1$$

Simplyfied calculation:
Real interest rate = Nominal interest rate – Inflation rate

FV, PV and IRR example

- You have got 100€. You have got the opportunity to invest it for 1 year and you are going to receive 110€.
- If the required rate of return is 6%, would you accept the project or would you reject it?



$$FV_1 = C_0 * (1+r)^t$$

$$PV = \frac{C_1}{(1+r)^t}$$

$$IRR = \frac{C_1}{C_0} - 1$$

$$r = \sqrt[t]{\frac{FV}{C_0}} - 1$$

$$t = \frac{ln\frac{FV}{C_0}}{\ln(1+r)}$$

FV, r_{eff}, example with compounded interest

- You have got €100. You put it in a 1Y deposit at a rate of 3%. Frequency of interest payment can be 1 year, 0.5 year, a quarter, a month. What are the Future Values of this deposit in those cases, if the deposit rate remains the same?
- What is the best option?

$$FV = C_0 * \left(1 + \frac{r}{f}\right)^{t*f}$$

$$r_e = \left(1 + \frac{r}{f}\right)^{f} - 1$$

$$\lim_{f \to \infty} \left(1 + \frac{1}{f}\right)^{f} = e^{r}$$

NPV example

You should invest 300€, and the borrower promise to pay 400€ in 4 years. The required rate of this investment is 10%.

Should you accept the offer, or should you reject it?

$$NPV = -C_0 + \frac{C_t}{(1+r)^t}$$

$$IRR = \sqrt[t]{\frac{FV}{C_0}} - 1$$

NPV example 2.

You invest €1000, and the borrower promise to pay €300 in 1 year, €400 in 2 years, €500 in 3 years. If the required rate is 20%, should you accept or reject the investment opportunity?

$$NPV = -C_0 + \sum_{t=1}^{n} \frac{C_t}{(1+r)^t}$$

As the above NPV formula is an n^{th} degree equation, the IRR can be approximated by an iterative method, or with Excel's IRR function (or with Solver)

Future value of a lump sum

Lump sum = a single payment made at a particular time

You have got €100. You put it in a 1Y deposit at a rate of 3%. Frequency of payment is semiannually.

What is the future value of this deposit?

What is the APR of this product?

$$FV = C_0 * \left(1 + \frac{r}{f}\right)^{t*f}$$

$$r_e = \left(1 + \frac{r}{f}\right)^{f} - 1$$

FV example – interest compounded at infinite frequency

You have got **€100**. You put it in a **1Y** deposit at a rate of **3%**. Frequency of payment is infinite. What is the future value of this deposit?

$$\boxed{FV = C_0 * e^{r*t}}$$

$$\boxed{r_e = e^r - 1}$$

The rule of 72...

The ‚Rule of 72' is a useful calculation method to approximate how long it will take to double your investment at a particular interest rate *(the interest is yearly compounded)*:

$$Years\ to\ double\ money\ (n) = \frac{72}{r}$$

$$Interest\ rate\ to\ double\ money\ in\ N\ years\ (r) = \frac{72}{n}$$

Notation:

- *r, (i)* : interest rate (%)
- *t, (n)* : time (in years)

Mixed interest calculations

When the duration of a deposit / investment is not an integer multiple of the interest period.

$$FV = C_0 * (1 + r * t_1) * \left(1 + \frac{r}{f}\right)^N * (1 + r * t_2)$$

$$r_{eff} = \left(\frac{FV}{C_0} - 1\right) * \frac{1}{t}$$

Notation:

- FV : future value of the investment,
- C_0 : principal amount,
- r : interest,
- N : # of full interest periods in the investment duration,
- f : # of compounding periods per year,
- t_1 : duration from the beginning of the investment until the first interest compounding (in year),
- t_2 : duration from the last compounding until the expiration day of the investment (in year).

Example - mixed interest calculation

You have got €100 000. You put it in a deposit at a rate of 4% on 10/04/2019. Frequency of interest payment is monthly (at the end of each month). The bank uses German-way of interest calculation. What is the future value of this deposit on 18/09/2019? What is the APR of this product?

$$\boxed{FV = C_0 * (1 + r * t_1) * \left(1 + \frac{r}{f}\right)^N * (1 + r * t_2)}$$

$$\boxed{r_{eff} = \left(\frac{FV}{C_0} - 1\right) * \frac{1}{t}}$$

Annuity - definition

An annuity is a series of equal payments made at equal intervals.

Examples of annuities are regular deposits to a savings account, monthly home mortgage payments, monthly insurance payments and pension payments.

The payments can be made daily, weekly, monthly, quarterly, semiannualy, yearly, or at any other regular interval of time.

- If the payments are made at the end of the time periods, so that interest is accumulated before the payment, the annuity is called an annuity-immediate, or ordinary annuity.
- An annuity-due is an annuity whose payments are made at the beginning of each period.

FV of an annuity

The formula without the extension is ‚at the end of the periods', with the extension ‚at the beginning of the periods'!

$$FV = c * \frac{(1+\frac{r}{f})^{t*f} - 1}{\frac{r}{f}} \left[* \left(1 + \frac{r}{f}\right) \right]$$

(By using the equation of for the sum of a geometric series: $S_n = a_1 \times \frac{q^n - 1}{q-1}$, where q=1+r)

PV of an annuity

The formula without the extension is ‚at the end of the periods', with the extension ‚at the beginning of the periods'!

$$PV = c * \frac{\left(1+\frac{r}{f}\right)^{t*f} - 1}{\frac{r}{f} * \left(1+\frac{r}{f}\right)^{t*f}} * \left[1+\frac{r}{f}\right]$$

Example - FV of annuity

You have got a baby, and you launch a baby bond scheme. Bank offers you a 3% inflation premium, length of scheme is 18 years, and monthly savings is €10. You started the saving at the beginning of the month. How much money can you widthdraw at the end of the 18th year in real term?

$$FV = c * \frac{(1+\frac{r}{f})^{t*f} - 1}{\frac{r}{f}} \left[* \left(1+\frac{r}{f}\right) \right]$$

Example - Personal loan

You would like to buy the new Apple iPhone 6plus for €1000. You hasn't got enough money, that's why you need to borrow the money. The terms of the personal loan is the following. Interest rate is 24% p.a., maturity is 1.5 year, frequency of payment is monthly (the beginning). If your scholarship is €150, are you able to buy this equipment or not?

$$PV = c * \frac{\left(1+\frac{r}{f}\right)^{t*f} - 1}{\frac{r}{f} * \left(1+\frac{r}{f}\right)^{t*f}} * \left[1+\frac{r}{f}\right]$$

Example - Home loan

Term of home loan is 15 years, and the lending rate is 6%, your parents give you €50.000, the price of flat is €200.000, frequency of payment is monthly (the end). If you can save the third of your €1.500 wage, are you able to buy this flat?

$$PV = c * \frac{\left(1+\frac{r}{f}\right)^{t*f} - 1}{\frac{r}{f} * \left(1+\frac{r}{f}\right)^{t*f}}$$

Bonus example - Building society

- Your monthly (at the beginning) saving is **20.000 HUF**. The term of saving is **4 years**. The amount of state subsidy is **30%** of annual saving up to **72.000 HUF** credited at the end of the year.
- What is the future value of this saving scheme and what is its APR rate?

$$FV_1 = C^1 * \left(1+\frac{r}{f}\right) * \frac{\left(1+\frac{r}{f}\right)^{t*f} - 1}{r/f}$$

$$FV_2 = C^2 * \frac{(1+r)^t - 1}{r}$$

Perpetuity - definition

Perpetuity refers to an infinite amount of time. In finance, perpetuity is a constant stream of identical cash flows with no end.

$$PV = \lim_{n\to\infty}\left[\frac{c}{(1+r)^1}+\frac{c}{(1+r)^2}+\cdots+\frac{c}{(1+r)^n}\right]=\frac{c}{1+r}\times\lim_{n\to\infty}\frac{\left(\frac{1}{1+r}\right)^n-1}{\frac{1}{1+r}-1}=c\times\frac{1}{r}$$

Growing Perpetuity - definition

A growing perpetuity is a series of periodic payments that grow at a proportionate rate and are received for an infinite amount of time. Examples of when the present value of a growing perpetuity formula may be used are shares, or commercial real estate.

$$PV = \lim_{n\to\infty}\left[\frac{c_1}{(1+r)^1} + \frac{c_1 \times (1+g)}{(1+r)^2} + \cdots + \frac{c_1 \times (1+g)^{n-1}}{(1+r)^n}\right] = \frac{c_1}{1+r} \times \lim_{n\to\infty} \frac{\left(\frac{1+g}{1+r}\right)^n - 1}{\frac{1+g}{1+r} - 1} = c_1 \times \frac{1}{r-g}$$

Bill of exchange - definition

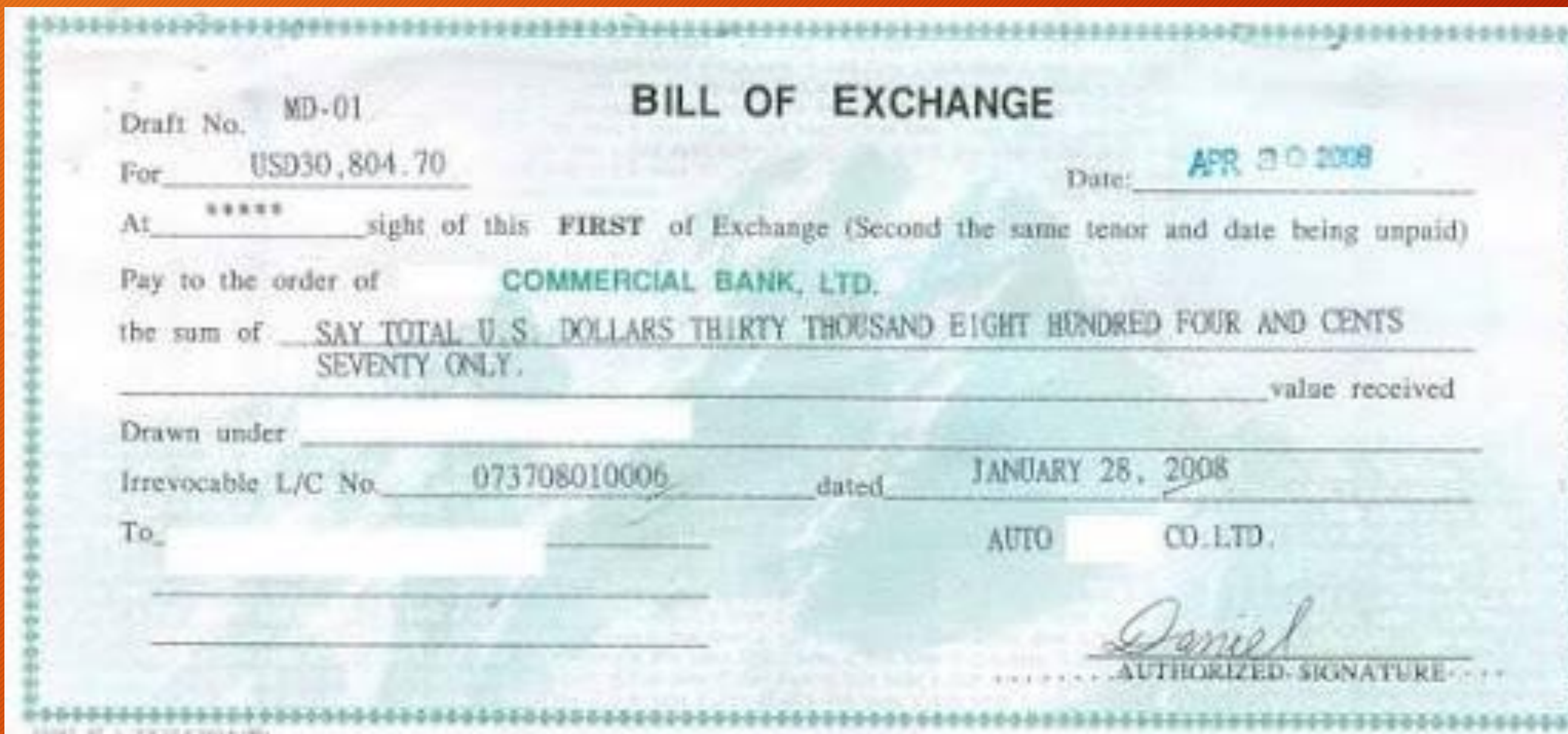
A bill of exchange is a written order that binds one party (payee) to pay a fixed sum of money to another party (drawee) on demand or at a predetermined date.

Bills of exchange are similar to checks and promissory notes. They can be drawn by individuals or banks and are generally transferable by endorsements. The difference between a promissory note and a bill of exchange is that the latter is transferable and can bind one party to pay a third party that was not involved in its creation.

There are up to three parties involved in a bill of exchange transaction. The drawee is the party that pays the sum specified by the bill of exchange. The payee is the one who receives that sum. The drawer is the party that obliges the drawee to pay the payee. The drawer and the payee are the same entity, unless the drawer transfers the bill of exchange to a third-party payee.

Bills of exchange generally do not pay interest, making them in essence post-dated checks. They may accrue interest if not paid by a certain date, however, in which case the rate must be specified on the instrument. They can conversely, be transferred at a discount before the date specified for payment.

Bill of exchange - sample



BILL OF EXCHANGE

Draft No. MD-01

For USD30,804.70

Date: APR 30 2008

At ***** sight of this FIRST of Exchange (Second the same tenor and date being unpaid)

Pay to the order of COMMERCIAL BANK, LTD.

the sum of SAY TOTAL U.S. DOLLARS THIRTY THOUSAND EIGHT HUNDRED FOUR AND CENTS SEVENTY ONLY. value received

Drawn under

Irrevocable L/C No. 073708010006 dated JANUARY 28, 2008

To

AUTO CO.LTD.

AUTHORIZED SIGNATURE

Bill of exchange - Formulas

The discount price of a bill of exchange*:

$$PV = N - I = N - N \times d \times t = N \times (1 - d \times t)$$

The calculated discount rate:

$$d^* = \frac{i}{1 + i * t}$$

The calculated lending rate:

$$i^* = \frac{d}{1 - d * t}$$

The treshold time of a bill of exchange:

$$t = \frac{1}{d} - \frac{1}{i}$$

****We usually calculate with a 365-days-long year.***

Bill of exchange - Example

A bank want to discount a bill of exchange on 20th of October. The maturity of the security is 12th of December. Its nominal value is €10,000. The discount rate used by the bank is 4%. The lending rate is 4.3%. What will be the discount price of the bill of exchange? What is cheaper for the bank's client - to discount the bill or to raise a loan? What it the threshold period at the given lending and discounting rate?

$$d^* = \frac{i}{1 + i * t}$$

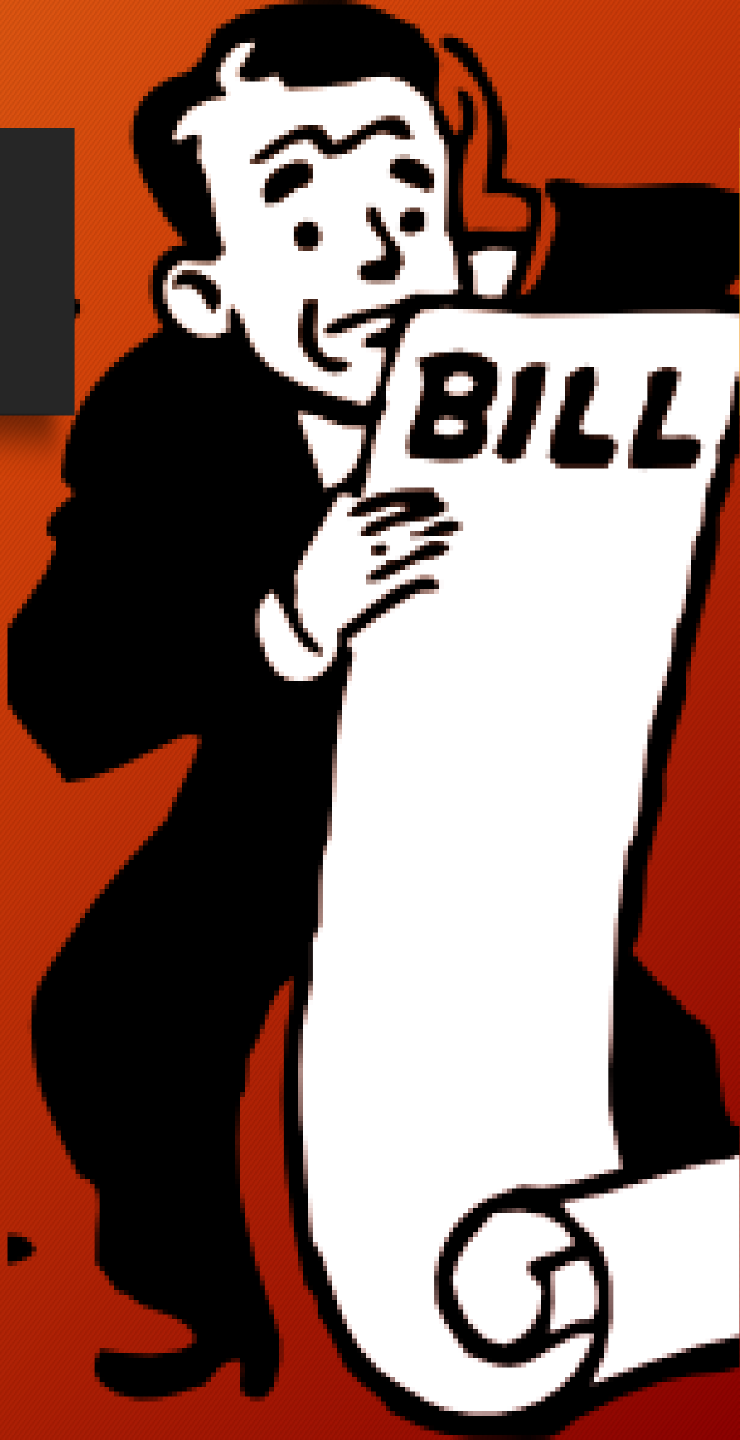
$$t = \frac{1}{d} - \frac{1}{i}$$

$$i^* = \frac{d}{1 - d * t}$$

$$PV = N * (1 - d * t)$$

T-bill - definition

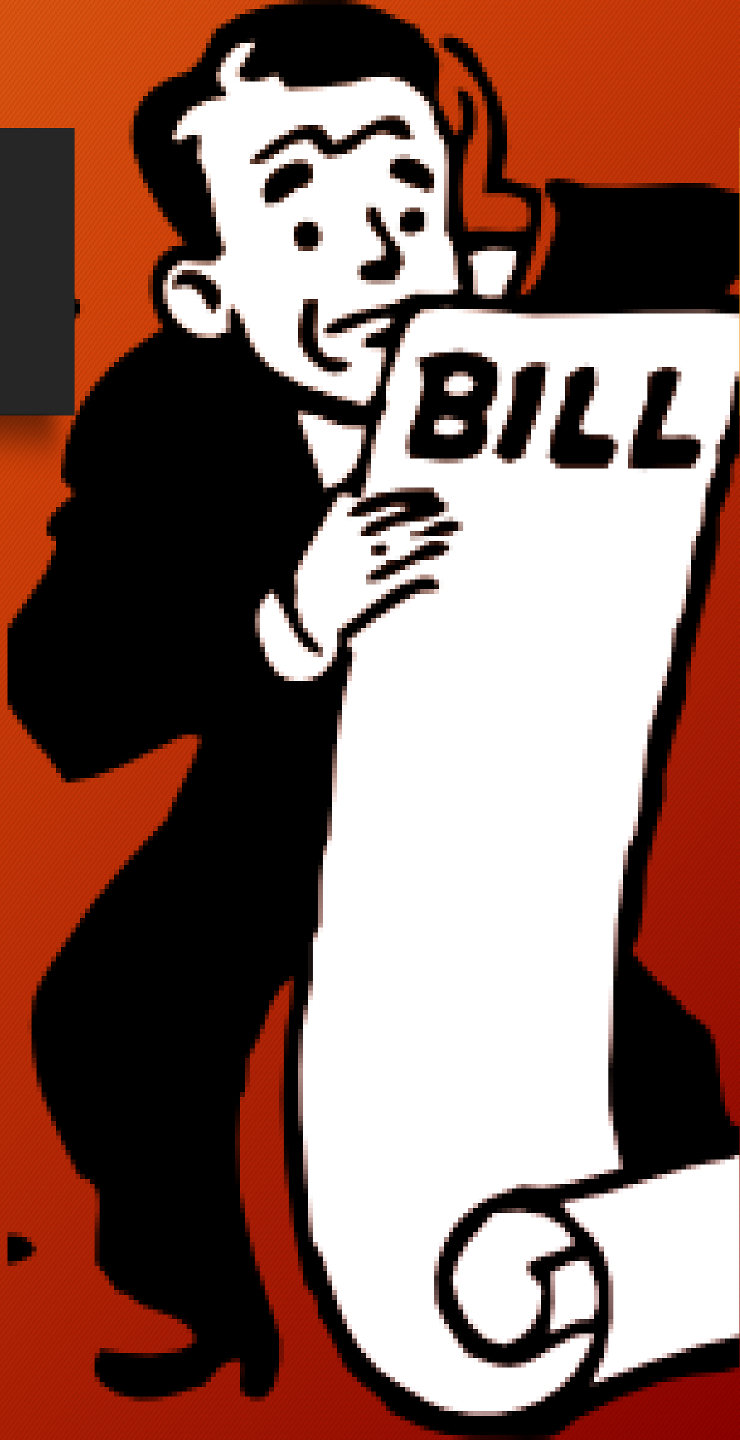
Short-term (usually less than one year, typically three months) maturity promissory note issued by a national (federal) government as a primary instrument for regulating money supply and raising funds via open market operations. Issued through the country's central bank, T-bills commonly pay no explicit interest but are sold at a discount, their yield being the difference between the purchase price and the par-value (also called redemption value).



T-bill - Formulas

$$PV = \frac{N}{1 + r * \frac{n}{360}}$$

$$IRR = \left(\frac{N - PV}{PV}\right) * \frac{360}{n}$$

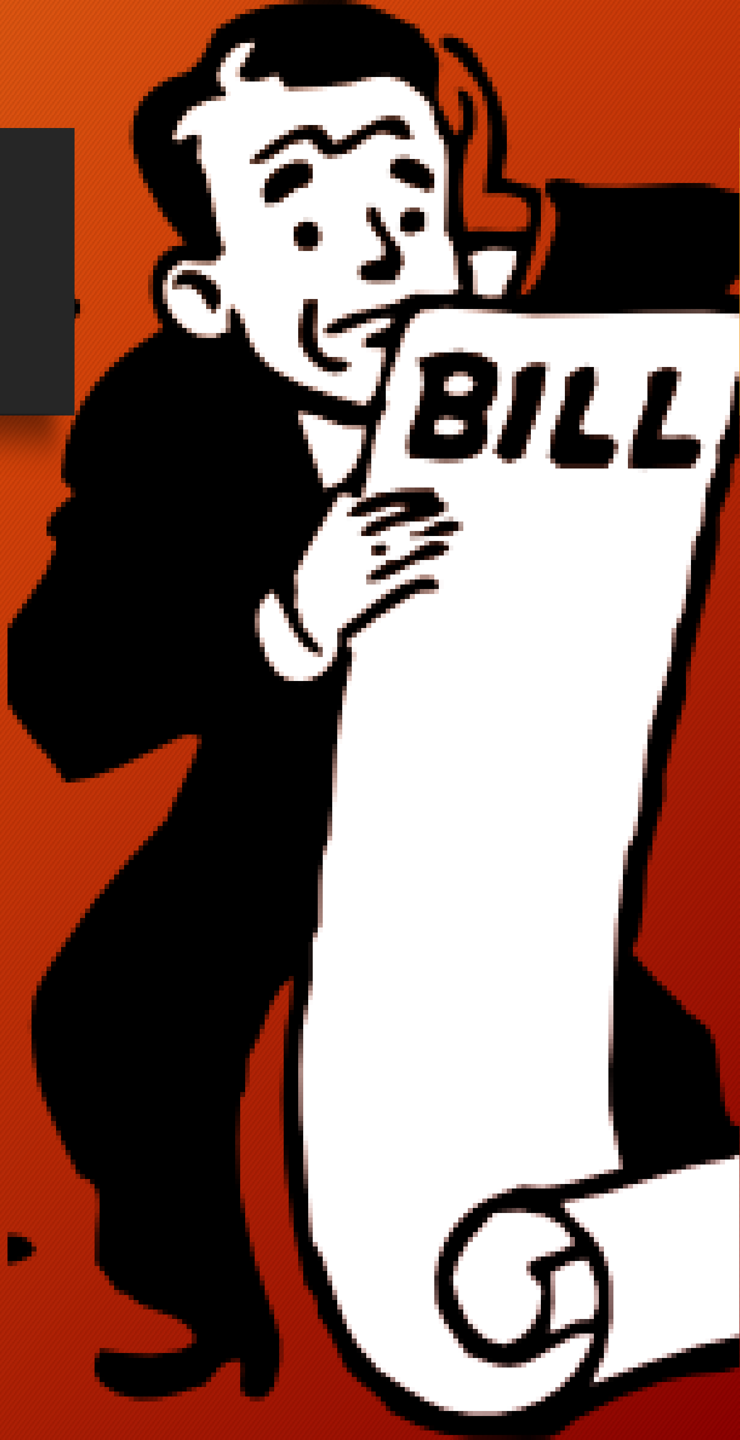


T-bill - Example

The maturity of a T-bill is 20 of March. Its price is now 98%. Would you buy this T-bill, if your required rate of return is 3%? What is the IRR of this bill (nominal yield)?

$$PV = \frac{N}{1 + r * \frac{n}{360}}$$

$$IRR = \left(\frac{N - PV}{PV}\right) * \frac{360}{n}$$



T-Bond definition

Long-term (maturity over 10 years) fixed interest rate debt security issued by a national (federal) government backed by its 'full faith and credit'. Next to treasury bills (maturity less than one year), and treasury notes (maturity one to ten years) T-bonds are the safest form of marketable investment.

T-Bond price calculation - Example 1.

You are considering to buy a T-bond, whose maturity is 5 years. The interest rate of the bond is 5%. The required rate of this asset is 7%. What is the maximum amount which you should sacrify to buy this bond? If the price of the bond is 110% of the nominal value, should you buy or not?

$$PV = c * \frac{(1+r)^t - 1}{r*(1+r)^t} + N * \frac{1}{(1+r)^t}$$

T-Bond price calculation - Example 2.

2021/J bond promises to pay 7% interest per annum (on every 23th of May until maturity). The next interest payment occurs on 23/05/2019. The required rate of return is 4%. What is the maximum price of this bond? Shall you buy or sell this bond (today: 08/02/2019), if the price is 110%?

$$PV = c * \frac{(1+r)^n - 1}{r * (1+r)^n} + N * \frac{1}{(1+r)^n}$$

$$X = \frac{c}{t}$$

Share pricing with constant dividend - Example

The next day the GDF Suez will pay 400€ dividend. You expect the same annual dividend for a very long time. The expected return is 10%. What is the maximum price of this share?

If the company pays out the dividend, what will be the maximum price?

$$PV = \frac{Div_1}{r}$$

Gordon model - Example

The Apple share costs $200. The next dividend payment on 6th of June 2019 will be $20. The growth rate is estimated to 10%. The required rate is 15%. What is the net price? What is the gros price? Shall you buy the share?

$$PV = \frac{Div_1}{r - g}$$

Perpetuity - Example

American Treasury issued a perpetuity with the following terms: coupon rate is 5%. The interest is paid in 4 of July. What is the value (gros price) of this perpetuity, if your required rate is 0.25%?

$$PV = \frac{Div_1}{r}$$

Thank you for your attention!

Questions?